

Description Logic

Summer Semester 2022

Exercise Sheet 2

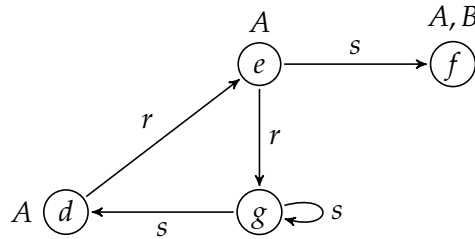
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Exercise 2.1 Consider the ABox

$$\mathcal{A} = \{d:A, e:A, f:A, f:B, (d,e):r, (e,g):r, (e,f):s, (g,g):s, (g,d):s\}$$

with the following graphical representation:



For each of the following concepts C , list all individuals that are instances of C w.r.t. $\mathcal{A}$. Compare your results to Exercise 1.2.

- (a) $A \sqcup B$
- (b) $\exists s. \neg B$
- (c) $\forall s. A$
- (d) $(\geq 2 s)$
- (e) $\exists s. \exists s. \exists s. \exists s. A$
- (f) $\forall s. \neg. \exists s. \exists s. \exists s. A$
- (g) $\neg \exists r. (\neg A \sqcap B)$
- (h) $\exists s. (A \sqcap \forall s. \neg B) \sqcap \neg \forall r. \exists r. (A \sqcup \neg A)$

Exercise 2.2

- (a) Extend the TBox $\mathcal{T}$ from Exercise 1.3 to a knowledge base $\mathcal{K}$ by adding axioms that capture the following statements (using the additional concept name *Broken*, and the individual names *Bob* and *QE2*):
 - (i) Cars have between three and four wheels.
 - (ii) Bicycles have exactly two wheels.
 - (iii) A vehicle is controlled by at most one human.

- (iv) A thing with a broken part is broken.
 - (v) Bob controls a car with a wheel that has a broken axle.
 - (vi) Bob is a human.
 - (vii) Bob controls QE2.
 - (viii) QE2 is a vehicle that travels on water.
- (b) Which of the following concepts is satisfiable w.r.t. $\mathcal{K}$?
- (i) $\text{Boat} \sqcap \exists \text{hasPart.Wheel}$
 - (ii) $\text{Boat} \sqcap \exists \text{poweredBy.Engine}$
 - (iii) $\text{Car} \sqcap \text{Bicycle}$
 - (iv) $\text{Driver} \sqcap \text{Vehicle}$
 - (v) $\text{Driver} \sqcap \text{Child}$
 - (vi) $\exists \text{controls.Car} \sqcap \text{Child}$
 - (vii) $\exists \text{controls.Car} \sqcap \text{Child} \sqcap \text{Human}$
- (c) Which of the following statements about $\mathcal{K}$ is true?
- (i) $\mathcal{K}$ is consistent.
 - (ii) Bob is an instance of Adult w.r.t. $\mathcal{K}$.
 - (iii) Bob is an instance of Driver w.r.t. $\mathcal{K}$.
 - (iv) Bob is an instance of $(\text{Adult} \sqcap \text{Driver})$ w.r.t. $\mathcal{K}$.
 - (v) Bob is an instance of $\exists \text{controls.}(\text{Car} \sqcap \text{Broken})$ w.r.t. $\mathcal{K}$.
 - (vi) QE2 is an instance of Boat w.r.t. $\mathcal{K}$.
 - (vii) Driver is subsumed by Human w.r.t. $\mathcal{K}$.
 - (viii) Adult is subsumed by Human w.r.t. $\mathcal{K}$.
 - (ix) $\text{Human} \sqcap \exists \text{controls.}(\text{Vehicle} \sqcap \exists \text{hasPart.Wheel} \sqcap \exists \text{poweredBy.Engine})$ is subsumed by Adult w.r.t. $\mathcal{K}$.
 - (x) $\exists \text{controls.Car}$ is subsumed by Adult w.r.t. $\mathcal{K}$.

Exercise 2.3 Consider the following acyclic TBox $\mathcal{T}$:

$$\begin{aligned}
 \{ & \text{MastersStudent} \equiv \text{Student} \sqcap \exists \text{enrolledIn.MastersProgram}, \\
 & \text{DiplomaStudent} \equiv \text{Student} \sqcap \exists \text{enrolledIn.DiplomaProgram}, \\
 & \text{Student} \equiv \text{Human} \sqcap \exists \text{attends.Course}, \\
 & \text{Lecturer} \equiv \text{Human} \sqcap \exists \text{teaches.Course}, \\
 & \text{Course} \equiv \text{Lecture} \sqcup \text{Seminar}, \\
 & \text{Human} \equiv \text{Woman} \sqcup \text{Man} \}
 \end{aligned}$$

and the following ABox $\mathcal{A}$:

$\{ \text{mary}:\text{MasterStudent}, \quad \text{tom}:\text{DiplomaStudent}, \quad \text{drSmith}:\text{Lecturer} \}$

Compute the expanded version $\hat{\mathcal{A}}$ of $\mathcal{A}$ w.r.t. $\mathcal{T}$.

Exercise 2.4 Proposition 2.7 from the lecture states that:

For every acyclic TBox $\mathcal{T}$ we can effectively construct an equivalent acyclic TBox $\hat{\mathcal{T}}$ such that the *right-hand sides* of concept definitions in $\hat{\mathcal{T}}$ contain *only primitive concepts*.

To prove this proposition, an expansion process (described below) that transforms $\mathcal{T}$ into $\hat{\mathcal{T}}$ was presented in the lecture.

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1:  $\mathcal{T}_0 := \mathcal{T}; i := 0;$ 
2: while a defined concept occurs on the right-hand side of a definition in  $\mathcal{T}_i$  do
3:   choose  $A \equiv C \in \mathcal{T}_i$  such that a defined concept  $B$  occurs in  $C$ ;
4:   let  $B \equiv D$  be the definition of  $B$  in  $\mathcal{T}_i$ ;
5:   replace all occurrences of  $B$  in  $C$  by  $D$ , and let  $\hat{C}$  be the obtained concept description;
6:    $\mathcal{T}_{i+1} := (\mathcal{T}_i \setminus \{A \equiv C\}) \cup \{A \equiv \hat{C}\};$ 
7:    $i := i + 1;$ 
8: end while
9:  $\hat{\mathcal{T}} := \mathcal{T}_i;$ 

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Complete the proof of Proposition 2.7. More precisely, show that this expansion process always terminates and the TBox $\hat{\mathcal{T}}$ is equivalent to $\mathcal{T}$.

Exercise 2.5 Recall Corollary 2.8 from the lecture, which shows that every primitive interpretation has a unique extension to a model of a given acyclic TBox. Consider the *cyclic* TBox

$$\mathcal{T} := \{A \equiv \exists r.B, B \equiv \exists r.\neg A\}.$$

- (a) Is there a primitive interpretation that cannot be extended to a model of $\mathcal{T}$?
- (b) Is there a primitive interpretation that can be extended in several ways to a model of $\mathcal{T}$?